I Semester M.Sc. Degree Examination, January/February 2018
(CBCS Scheme)
MATHEMATICS
M101T : Algebra – I

Time : 3 Hours

Max. Marks : 70

Instructions : *1) Answer **any 5** questions.*
*2) **All** questions carry **equal** marks.*

1. a) Let $\phi : G \to \overline{G}$ be an epimorphism with Kernel K and let N be a normal subgroup of G. Then prove that $\frac{G/K}{N/K} \approx G/N$.

   b) Show that $T : G \to G$ defined by $T(x) = x^{-1}$ is an automorphism if and only if G is abelian.

   c) State and prove the Cayley's theorem for finite groups. (5+4+5)

2. a) State and prove the orbit-stabilizer theorem.

   b) Derive the class equation for finite groups.

   c) Define a p-group. If G is a finite group of prime power order. Prove that G has a non-trivial center. (5+5+4)

3. a) State and prove the Sylow first theorem.

   b) Let $O(G) = pq$, where p and q are distinct primes with $p < q$ and $q \not\equiv 1 (\text{mod } p)$. Then prove that G is abelian and cyclic. (8+6)

4. a) Define a solvable group. Prove that every subgroup of a solvable group is solvable.

   b) State and prove the Jordan-Holder theorem.

   c) Show that symmetric group S_4 is solvable, but not solvable. (4+7+3)

5. a) If R is a ring with unity in which (0) and R are the only two left ideals, then prove that R is a divison ring.

   b) If U is an ideal of a ring R, let $[R : U] = \{x \in R : rx \in U \ \forall \ r \in R\}$. Prove that $[R : U]$ is an ideal of R containing U.

   c) Let R and R′ be rings and ϕ is a homomorphism of R onto R′ with Kernel U. Then show that $R' \approx R/U$. **(5+4+5)**

6. a) Define a principal ideal and principal ideal ring. Prove that every field is a principal ideal ring.

   b) Define maximal ideal of a ring. If R is a commutative ring with unit element and M is an ideal of R, then show that M is a maximal ideal of R if and only if R/M is a field.

   c) Prove that in a principal ideal ring, every non-zero prime ideal is maximal ideal. **(5+6+3)**

7. a) Define an euclidean ring. Let $x = a + ib$, $y = c + id$ be any two elements in $Z[i] - \{0\}$ then prove that it is an euclidean ring.

   b) Show that every Euclidean ring is a principle ideal ring.

   c) State and prove the unique factorization theorem. **(5+4+5)**

8. a) Prove that deg(fg) = deg(f) + deg(g) for f, G ∈ R[x]. Further, if R is an integral domain, then show that R[x] is also an integral domain.

   b) Show that the product of two primitive polynomials is a primitive polynomial.

   c) Verify that $f(x) = x^3 + x^2 - 2x - 1 \in Q[x]$ is irreducible polynomial, by using Eisenstein criteria. **(5+5+4)**